Planteamiento e implementaciónn de un problema generalizado de autovalores con el algortimo de Von Mises

Francisco Javier 曹 (Tsao) Santín
tsao@enelparaiso.org

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Capítulo 1

Teoría numérica

Partimos de un problema de autovalores generalizado, de aspecto:

$$K \cdot \overline{v} = \lambda \cdot M \cdot \overline{v} \tag{1.1}$$

y disponemos de un algoritmo numérico para la obtención del máximo autovalor de un problema standard del tipo:

$$A \cdot \overline{u} = \lambda \cdot \overline{u} \tag{1.2}$$

que resolveremos iterando de esta manera: dado un $\overline{u}_0$ tal que

$$\overline{u}_0 \cdot \overline{u}_0^t = 1 \tag{1.3}$$

hacemos:

$$\overline{u}_{k+1} = \frac{A \cdot \overline{u}_k}{|A \cdot \overline{u}_k|} \tag{1.4}$$

$$\rho(\overline{u}_{k+1}) = \overline{u}_{k+1}^t \cdot A \cdot \overline{u}_{k+1} = \overline{u}_{k+1}^t \cdot \overline{w}_{k+1} \tag{1.5}$$

Para poder llegar a este método precisamos de un cambio de base: supongamos que:

$$M = L \cdot L^t \tag{1.6}$$

entonces podemos relacionar:

$$\overline{u} = L^t \cdot \overline{v} \tag{1.7}$$

además:

$$A = L^{-1} \cdot K \cdot L^{-1t} \tag{1.8}$$

y así seguimos usando los arrays del problema generalizado ($\overline{v}$,K,M). Sustituyendo sobre (4):

$$L^t \cdot \overline{v}_{k+1} = \frac{L^{-1} \cdot K \cdot L^{-1t} \cdot L^t \cdot \overline{v}_k}{|L^{-1} \cdot K \cdot L^{-1t} \cdot L^t \cdot \overline{v}_k|} = \frac{L^{-1} \cdot K \cdot \overline{v}_k}{|L^{-1} \cdot K \cdot \overline{v}_k|} \tag{1.9}$$

Si renombramos el numerador:

$$L^{-1} \cdot K \cdot \overline{v}_k = \overline{z}_{k+1} \tag{1.10}$$

y recordando $\overline{w} = K \cdot \overline{v}$, tenemos:

$$L^t \cdot \overline{v}_{k+1} = \frac{L^{-1} \cdot \overline{w}_k}{|\overline{z}_{k+1}|} \tag{1.11}$$

Claro que no queremos obtener las inversas de L, resultado de factorizar M, ya que es una operación costosa. Luego resolvemos primero el sistema:

$$L \cdot \overline{z}_{k+1} = \overline{w}_k \tag{1.12}$$

y a continuación, nombrando $\overline{v}_{k+1} \cdot |\overline{z}_{k+1}| = \overline{y}_{k+1}$, resolvemos:

$$L^t \cdot \overline{y}_{k+1} = \overline{z}_{k+1} \tag{1.13}$$

Y finalmente:

$$\overline{v}_{k+1} = \frac{\overline{y}_{k+1}}{|\overline{z}_{k+1}|} \tag{1.14}$$

o también:

$$\overline{v}_{k+1} = \frac{\overline{y}_{k+1}}{|\overline{y}_{k+1}|} \tag{1.15}$$

Pero cuidado: si nos valemos de $\overline{z}$ la normalización la realizamos sobre la matriz identidad, sin embargo si utilizamos $\overline{y}$ debemos normalizar sobre la matriz M, ya que de (3) deducimos:

$$\overline{v}^t \cdot M \cdot \overline{v} = 1 \tag{1.16}$$

El cociente de Rayleigh:

$$\rho_{k+1} = \overline{v}_{k+1}^t \cdot K \cdot \overline{v}_{k+1} = \overline{v}_{k+1}^t \cdot \overline{w}_{k+1} \tag{1.17}$$

Capítulo 2

Implementación

Bien, teniendo clara la teoría, no nos resulta demasiado difícil plantear el programa que resuleve nuestro problema de autovalores. Hay que factorizar la matriz M, en primer lugar. Suponiendo que M sea simétrica (en caso contrario la solución se sale del rango de lo real), almacenaremos obviamente sólo la mitad de M, y L. Después arrancamos la iteración calculando un vector $\overline{v}_0$ de componentes aleatorias, lo normalizamos respecto de M y calculamos $\overline{w}_0$ y ρ_0. Y a partir de ahí, con k desde 1 hasta convergencia:
-$L \cdot \overline{z}_{k+1} = \overline{w}_k$ (de (12)) obtenemos $\overline{z}$ con un algoritmo de resolución de sistemas triangulares inferiores
-$L^t \cdot \overline{y}_{k+1} = \overline{z}_{k+1}$ (de (13)) obtenemos $\overline{y}$ con un algoritmo de resolución de sisemas triangulares superiores
-normalizamos $\overline{y}_{k+1}$, obteniendo por tanto $\overline{v}_{k+1}$
-obtenemos nuevo cociente de Rayleigh
-comparamos los nuevos ρ con los anteriores, y establecemos convergencia según precisión. Si ésta se produce, aceptamos los últimos valores y detenemos el bucle. En caso contrario, almacenamos los últimos valores de ρ y $\overline{v}$ para comparar, y $\overline{w}$ para generar la siguiente iteración, sobre los anteriores y volvemos a ejecutar otro ciclo.

Finalmente, si queremos afinar un poco más, podremos ahorrar un poco de memoria si almacenamos L sobre M, pero para ello tendremos que previamente normalizar $\overline{v}_0$ con M, y en pasos sucesivos normalizar $\overline{z}$ sobre la matriz identidad, lo que nos obliga a escribir una subrutina más. Si el problema es extremadamente grande, debemos optar por ésto.

Capítulo 3

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